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Delay and Peak-Age-of-Information of ALOHA Networks with Limited Retransmissions

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***Abstract*—This letter analyzes the impact of packet retransmissions on the performance of wireless random access protocols. In particular, we focus on the effects of two controllable parameters, namely the rate at which retransmissions occur and the number of allowed retransmissions. In a scenario where packets are generated according to a Poisson process and new packets substitute old ones upon arrival, we determine the conditions in which a given throughput or packet success probability constraint is achievable. We also prove, for a general packet success probability function of the traffic, that selecting the smallest feasible value of the number of allowed retransmissions along with an appropriate retransmission rate is optimal in terms of mean delay to transmit packets and mean *Peak-Age-of-Information*. We illustrate this result with numerical examples.**

***Index Terms*—Wireless, retransmission, delay, AoI.**

I. Introduction

The intrinsic characteristics of wireless transmissions and the specific policies of spectrum use based on frequency bands have led to several theoretical and practical challenges for the deployment of wireless systems. Broadband services in licensed spectra can rely on non-conflicting access to a communication resource over an extended period. In networks employing unlicensed frequencies, random access allows spectrum sharing [1]. In the emerging scenarios of massive connectivity for machine-type devices [2], random access is essential for allowing efficient access from a large population of devices with unpredictable and intermittent activity. In this case, the challenge is how to coordinate random transmissions so that each individual link has an acceptable performance.

With the increase in the number of wirelessly connected devices, there is a growing interest in ALOHA-based protocols, either in more practical domains considering Low Power Wide Area Networks (LPWAN) (e.g., [3]) or more fundamental studies of their limits (e.g., [4]). This paper focuses on the latter motivated by still open questions about the fundamental performance limits of ALOHA networks with a limited number of packet retransmissions [5], [6]. Retransmissions have a positive effect of increasing the reliability of transmissions by decreasing the probability that a packet is lost because packets detected in error could be retransmitted up to a given value. On the contrary, if more retransmissions are allowed, the network will be subjected to a greater traffic because packets will "stay longer" in the system. An increase in traffic leads to a greater chance of collision, and thus, a decrease in the probability that a packet is successfully received at its first transmission.

In this paper, we analyze the impact of a limited number of retransmissions in the network performance by tuning the number of allowed retransmissions $m \in \mathbb{N}$ and the rate $r \in \mathbb{R}_+$ at which the packet to be retransmitted will access the network. Specifically, we consider a scenario where only the most recent event matters, and thus, a packet waiting to be retransmitted is dropped whenever a new packet arrives. In this scenario, in addition to delay, another metric of interest is the *Age-of-Information* (AoI) defined in [7], [8]. For a given user, at time t, let $U(t)$ be the time the last received packet was generated. Then, the *age* metric is defined as $\Delta(t) \triangleq t - U(t)$. Here we use a related metric called *Peak-Age-of-Information* (PAoI), which corresponds to the *Age-of-Information* when a packet is successfully received. The mean PAoI metric considers the time spent by dropped packets, which is an important difference from the classical mean delay metric.

We assume that the generation of packets is conditioned to events that follow a Poisson process. Thus, discarding a packet too soon (small m) may have a high cost in the PAoI metric because the node does not know when the next packet is coming. Fig. 1 illustrates why increasing m decreases the mean PAoI in this case. However, we cannot forget that increasing m increases interference, which may increase the PAoI in the end. Our aim in this paper is to select the optimal pair r and m for the metrics throughput, delay, and PAoI.

The main differences of our paper from recent works is that we deal with a general transmission success probability function of the traffic, i.e., this can represent the collision model, the capture model, or any arbitrary model that does not depend directly on r and m. Also, we consider limiting the maximum number of retransmissions to improve performance. In this framework, we prove that selecting the smallest number of allowed retransmissions that satisfy some requirements is the optimal approach. In [9]–[11] the transmission success probability is derived from the classical collision model; in [9] the authors propose a random access scheme to optimize the AoI for a set of arrival rates and a large number of nodes; in [10] the authors propose a random access scheme to optimize the AoI based on the instantaneous AoI of each node; in [11] the authors show that random access perform worse than scheduled access with feedback by a factor of approximately $2e$ in the AoI.

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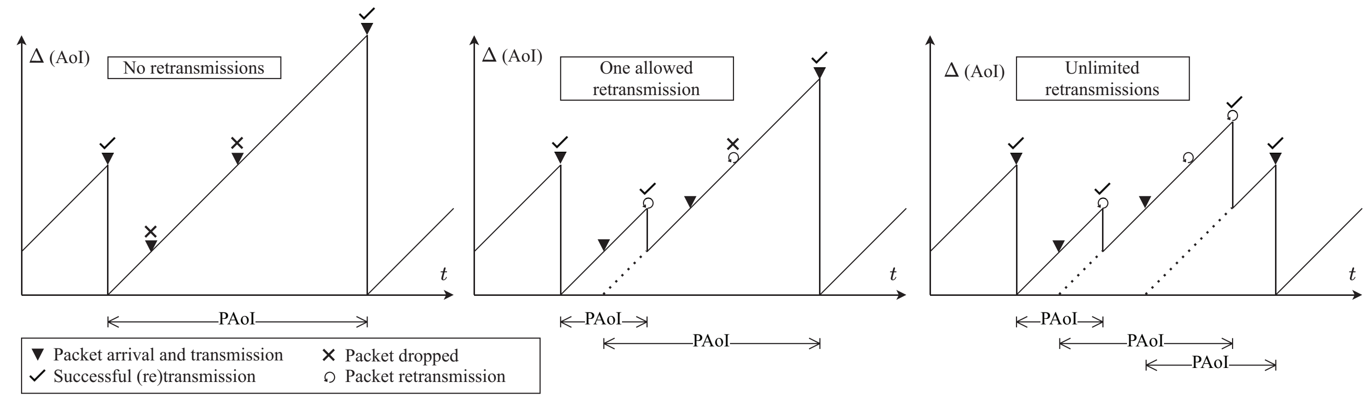

Figure 1. Examples of measures of the PAoI metric for different retransmission strategies.

II. System Model

The network has a given number of nodes (possibly infinite), each node receives a packet whenever an event associated with that node occurs, and we suppose the events occur according to a Poisson process of parameter $a > 0$. Thus, a is also the arrival rate of packets per node. Once the packet arrives, the node transmits the packet[1] to a receiver. If the packet is successfully transmitted, it is dropped[2]. Otherwise, the node transmits it again after an exponentially distributed time with a parameter $r > 0$, which is the retransmission rate. In this scenario, we suppose that only the data related to the most recent event matter, i.e., if a new packet arrives, the old one waiting to be retransmitted is dropped. Another case where the packet is also dropped is when it reaches the number of allowed retransmissions $m \in \mathbb{Z}_+$. To attain analytical tractability of the system model we further assume:

- stationary homogeneous traffic λ per unit of area, i.e., all receivers experience interference with the same distribution[3];
- transmission time is negligible compared with the waiting time to retransmit or for a new packet to arrive in the node;
- stationary success probability of a single transmission is a general function $p_s : \mathbb{R}_+ \longrightarrow [0, 1]$ of traffic λ that satisfies the conditions of the theorem presented below.

Theorem 1. *Let $p_s : \mathbb{R}_+ \longrightarrow (0, 1]$ be a continuously differentiable function and $\psi(\lambda) \triangleq -\lambda \frac{\mathrm{d}}{\mathrm{d}\lambda} \ln(p_s(\lambda))$. If ψ is a monotonous increasing function and*

$$\lim_{\lambda \to \infty} \psi(\lambda) > 1, \tag{1}$$

then the throughput $\mathcal{T}(\lambda) \triangleq \lambda p_s(\lambda)$ has a unique local maximum, which is a global maximum.

Proof. See Appendix A. □

[1]This can be easily modified to transmit the packet after an exponentially distributed time upon arrival without compromising the analytical tractability.

[2]Successful reception acknowledgment is transmitted in an error-free channel.

[3]The main purpose of this assumption is to eliminate the intricate coupling and interactions between the transmitting nodes. This is valid in large-scale networks with high-mobility nodes [12], with a small access probability [13] or with frequency hopping in several channels.

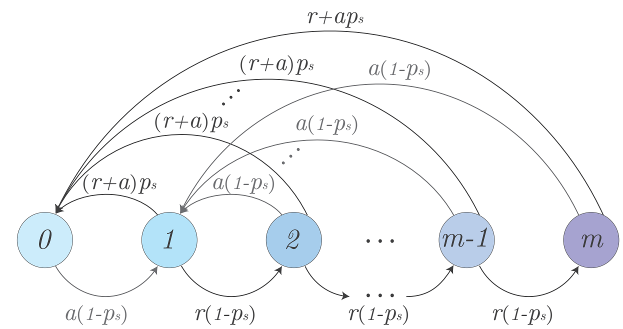

Figure 2. Markov chain transition rates for a typical node.

From now on, let us use p_s to denote $p_s(\lambda)$ to lighten the notation. The state of a typical node can be represented as a continuous-time Markov chain with transition rates:

$$\begin{aligned} k \to k+1 &: r\,(1-p_s), && 0 < k < m, \\ k \to 1 &: a\,(1-p_s), && 0 \le k \le m, \\ k \to 0 &: (r+a)\,p_s, && 0 < k < m, \\ m \to 0 &: r + a\,p_s, \end{aligned}$$

where the state $k \in \{1, 2, \ldots, m\}$ represents the number of transmissions that occurred with the buffered packet, and $k = 0$ represents that the buffer is empty. These transition rates are illustrated in the diagram of Fig. 2. The nodes that are retransmitting have a transmission rate of $r + a$, because they can transmit either by retransmission or when a new packet arrives. On the other hand, the nodes that are not retransmitting ($k = 0$) have a transmission rate of a.

Let us define $\{\pi_k\}_{k \in \mathbb{Z}_+}$ as the stationary probabilities of the typical node being in the kth retransmission. Then, in the stationary state, the network traffic is given by

$$\begin{aligned} \lambda &= \frac{\lambda_0}{a}\left(a\pi_0 + (r+a)(1-\pi_0)\right) \\ &= \lambda_0 \left(1 + \frac{r}{a}(1-\pi_0)\right), \end{aligned} \tag{2}$$

where λ_0 is the traffic without retransmission, i.e., if $r = 0$, then $\lambda = \lambda_0$. It is important to remember that p_s depends on the traffic of the system, and thus, we have a challenging problem because λ also depends on p_s owing to the dependence on the stationary probabilities. The following section circumvents this problem and presents the analytical results of the paper.

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III. Analysis

Let us assume that the packet arrival rate a and the traffic without retransmissions λ_0 are fixed and known. Our objective here is to optimize the retransmission rate r and number of allowed retransmissions m in view of some performance metrics. We begin by solving the Markov chain of Fig. 2 for its stationary probabilities.

Proposition 1. *The stationary probability π_k of finding the typical node in the kth retransmission state is given by*

$$\pi_0 = 1 - \frac{a}{r}\frac{\rho\,(1-\rho^m)}{1-\rho},$$
$$\pi_k = \frac{a}{r}\,\rho^k, \qquad 1 \le k \le m,$$

where $\rho \triangleq \dfrac{r(1-p_s)}{r+a}$ is the retransmission probability.

Proof. See Appendix B □

Using Proposition 1, we can rewrite (2) in a simpler form

$$\lambda = \frac{1-\rho^{m+1}}{1-\rho}\lambda_0. \tag{3}$$

Let $\lambda^* \in \mathbb{R}_+$ be the traffic for which we have the maximum throughput $\mathcal{T}^* = \lambda^* p_s^*$, where $p_s^* = p_s(\lambda^*)$. Theorem 1 guarantees the existence and uniqueness of λ^*. The optimum traffic λ^* can be found by solving the equation $\psi(\lambda^*) = 1$, where ψ is defined in Theorem 1. We can achieve λ^* by adjusting the retransmission rate to an appropriate r^* for each $m \in \mathbb{Z}_+$. In general, the following lemma must be satisfied.

Lemma 1. *For all $r \in \mathbb{R}_+$, $m \in \mathbb{Z}_+$,*

$$1 \le \frac{\lambda}{\lambda_0} \le \frac{1-(1-p_s)^{m+1}}{p_s}.$$

Proof. See Appendix C. □

Therefore, to achieve the optimum traffic λ^* we must have

$$\frac{p_s^*\lambda^*}{1-(1-p_s^*)^{m+1}} < \lambda_0 \le \lambda^*. \tag{4}$$

There might exist several values for m that satisfy (4). Let us focus on the best choice in view of some performance metrics.

A. Stationary Mean Delay

Let D be the improper random variable that represents the delay of a typical packet. If it is dropped, then $D = \infty$.

From the Markov chain we can show that the PDF of D is

$$f_D(t) = p_s\delta_0(t) + \sum_{k=1}^{m} p_s\rho^k \frac{(r+a)^k t^{k-1}}{(k-1)!}\,\mathrm{e}^{-(r+a)t}, \tag{5}$$

for $t \in \mathbb{R}_+$, where δ_0 is the Dirac delta function.

Integrating f_D on $[0,\infty)$ gives the probability of not discarding the packet, i.e., the packet success probability

$$\mathbb{P}(D<\infty) = p_s\frac{1-\rho^{m+1}}{1-\rho}. \tag{6}$$

As expected, this satisfies $\lambda_0\mathbb{P}(D<\infty) = p\lambda = \mathcal{T}$, because it is another form of calculating the throughput.

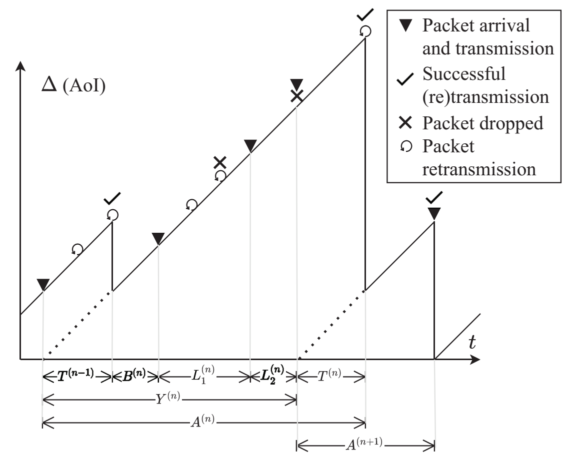

Figure 3. Time evolution of the *Age-of-Information* metric.

Then, the mean delay of a successfully transmitted packet, which we shall define as $\overline{D} \triangleq \mathbb{E}[D \mid D<\infty]$, is given by

$$\overline{D} = \frac{\int_0^\infty t\,f_D(t)\,\mathrm{d}t}{\mathbb{P}(D<\infty)} = \frac{1}{r+a}\left(\frac{1}{1-\rho} - \frac{1+m\rho^{m+1}}{1-\rho^{m+1}}\right). \tag{7}$$

B. Stationary Mean PAoI

The random variable PAoI $A^{(n)}$ measures the time between the nth successfully received packet transmission and the previous successfully received packet generation, and is defined as

$$A^{(n)} = Y^{(n)} + T^{(n)}, \tag{8}$$

where $Y^{(n)}$ is the interarrival time between packets of a typical node that are not dropped, and $T^{(n)}$ is the time a packet that is not dropped spends in the system, i.e., $\mathbb{E}[T^{(n)}] = \overline{D}$.

Fig. 3 shows a realization of the stochastic process with $m = 2$ and two dropped packets.

The random variable $Y^{(n)}$ is composed by

$$Y^{(n)} = T^{(n-1)} + B^{(n)} + \sum_{i=1}^{K^{(n)}} L_i^{(n)}, \tag{9}$$

where B follows an exponential distribution of the parameter a and represents the packet arrival time since the last successful transmission (memoryless property), K follows a geometric distribution of the parameter $\mathbb{P}(D<\infty)$ and represents the number of dropped packets until the arrival of a packet that is not dropped, and L_i is the time spent by the ith dropped packet since its arrival until a new packet arrives.

A closed-form expression for the mean PAoI metric is given by the following proposition.

Proposition 2. *The stationary mean PAoI is given by*

$$\mathbb{E}[A] = \frac{1}{a} + \frac{1}{a+r}\frac{1-\rho^m}{1-\rho^{m+1}}\left(\frac{1-p_s}{p_s} + \frac{\rho}{1-\rho} - \frac{m\rho^{m+1}}{1-\rho^m}\right) + \frac{1}{a}(1-p_s)\,\rho^m\left(\frac{1-\rho}{p_s\,(1-\rho^{m+1})} - 1\right). \tag{10}$$

Also, $\displaystyle\lim_{m\to\infty}\mathbb{E}[A] = \frac{1}{a} + \frac{1}{a+r}\left(\frac{1-p_s}{p_s} + \frac{\rho}{1-\rho}\right).$

Proof. See Appendix D. □

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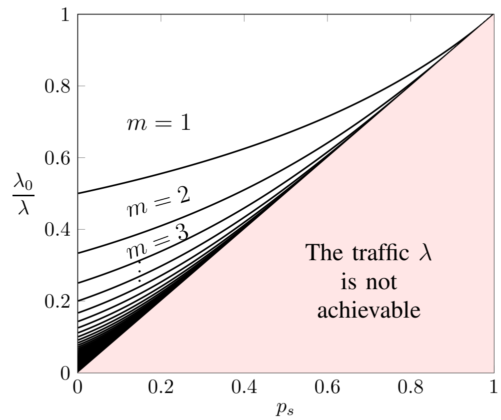

Figure 4. Best choice of m for a given traffic λ.

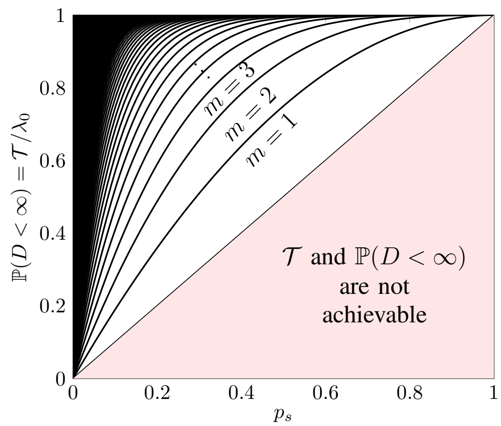

Figure 5. Best choice of m for a given throughput $\mathcal{T}$ or packet success probability $\mathbb{P}(D < \infty)$.

C. Retransmission Optimization

Finally, we can state the main result of the present paper.

Proposition 3. *Let $\lambda \in \mathbb{R}_+$. If $\lambda_0 \in (\lambda p_s(\lambda), \lambda)$, then there exist infinite pairs of retransmission rates $r \in \mathbb{R}_+$ and number of allowed retransmissions $m \in \mathbb{Z}_+$ that achieve the network traffic λ. Furthermore, the pair with the smallest m has the minimal mean delay $\overline{D}$ and mean PAoI $\mathbb{E}[A]$.*

Proof. See Appendix E. □

Given a requirement (constraint) among the metrics throughput $\mathcal{T}$, network traffic λ, and packet success probability $\mathbb{P}(D < \infty)$, we should use the smallest number of allowed retransmissions m such that we can adjust the retransmission rate r to achieve the requirement and satisfy Lemma 1. Then, according to Proposition 3, we minimize both the metrics mean delay $\overline{D}$ and mean PAoI $\mathbb{E}[A]$.

Fig. 4 shows the regions of the $\lambda_0/\lambda \times p_s$ plane for which we can choose the best value of the maximum number of retransmissions m when the requirement is a given traffic λ. Fig. 5 is similar to Fig. 4, but now for the $\mathcal{T}/\lambda_0 \times p_s$ plane.

If $p_s^*\lambda^* < \lambda_0 \leq \lambda^*$ we can choose as requirement the maximum throughput $\mathcal{T}^*$ or, equivalently, the optimum traffic λ^* and, then, use the figures to find the best value for m. The following section provides a numerical example.

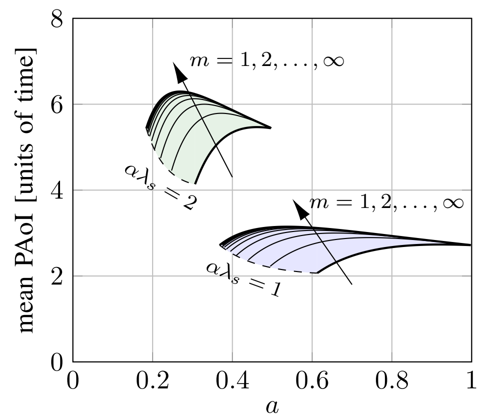

Figure 6. Mean PAoI $\mathbb{E}[A]$ as a function of a for the maximum $\mathcal{T}$.

IV. Numerical Example

Let us study a simple scenario where the packet success probability function is $p_s(\lambda) = \mathrm{e}^{-\alpha\lambda}$, for some parameter $\alpha > 0$. Then, it is easy to show that the maximum throughput is achieved at $\lambda^* = 1/\alpha$. Then, $p_s^* = 1/\mathrm{e}$ and $\mathcal{T}^* = (\alpha\mathrm{e})^{-1}$. Let us consider that the traffic without retransmission is given by $\lambda_0 = a\lambda_s$, where λ_s is a quantity related to the number of users (e.g., the mean number of users per unit of area).

Then, we can plot the mean PAoI (Fig. 6) as a function of the rate of packets per user a, which is a fixed and known quantity in the original problem. This is done for each number of allowed retransmissions $m \in \mathbb{Z}_+$ that satisfies Lemma 1 with $\lambda = \lambda^*$, so that we can adjust the retransmission rate r^* and achieve the maximum throughput $\mathcal{T}^*$. The blue and green areas correspond to the optimum operating points when m varies from 1 to ∞. The green area is from a system that has twice as many users as the blue area system. As expected, Proposition 3 is observed in Fig. 6, i.e., the smaller the number of allowed retransmissions m, the smaller the mean PAoI. Further, we can see that as we increase m, the PAoI performance deteriorates. On the other hand, the region for which we can achieve the maximum throughput increases. For example, to achieve the maximum throughput when $a = 1/2$ and $\alpha\lambda_s = 1$, we need $m > 1$. More precisely, $m = 2$ from Fig. 4, since $\lambda_0/\lambda^* = 1/2$ and $p_s = 1/\mathrm{e} \approx 0.368$.

V. Conclusion

In this letter, we studied a single class random access wireless network with homogeneous traffic, general transmission success probability function, and a retransmission scheme where packets are dropped when new packets arrive at the node or the number of allowed retransmissions is reached. For a given throughput (or traffic or packet success probability) requirement, we showed that the optimal approach is to use the minimum number of allowed retransmissions such that, along with an appropriate retransmission rate, the desired requirement is achievable. This approach is optimal for two different metrics: mean delay and mean *Peak-Age-of-Information* (PAoI). This suggests that choosing the smallest number of allowed retransmissions that achieves the network requirements could be an interesting design heuristic.

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APPENDIX A
PROOF OF THEOREM 1

The derivative of the throughput at $\lambda \in \mathbb{R}_+$ is

$$\mathcal{T}'(\lambda) = p_s(\lambda)\left(1 - \lambda\left(\tfrac{-p_s'(\lambda)}{p_s(\lambda)}\right)\right) = p_s(\lambda)\,(1 - \psi(\lambda)).$$

Since p_s is a continuously differentiable function, $\mathcal{T}'$ is a continuous function. In addition, there exists $\lambda^* \in \mathbb{R}_+$ such that $\mathcal{T}'(\lambda^*) = 0$ by the Intermediate Value Theorem, because $\mathcal{T}'(0) = p_s(0) > 0$ and there exists a $\lambda_0 > 0$ such that $\mathcal{T}'(\lambda_0) < 0$, by (1). Then, using that ψ is a monotonous function, we have that if $\lambda < \lambda^*$, then $\mathcal{T}(\lambda)$ increases monotonically. Otherwise, if $\lambda > \lambda^*$, then $\mathcal{T}(\lambda)$ decreases monotonically, which concludes the proof.

APPENDIX B
PROOF OF PROPOSITION 1

From the Markov chain transition probabilities, we have that, for $2 \le k \le m$, $(r+a)\pi_k = r(1-p_s)\pi_{k-1}$. Thus, $\pi_k = \left(\frac{r(1-p_s)}{r+a}\right)^{k-1}\pi_1$. Using this relation along with

$$a(1-p_s)\pi_0 = (r+a)p_s \textstyle\sum_{k=1}^{m}\pi_k + r(1-p_s)\pi_m,$$
$$r(1-p_s)\pi_1 = (rp_s + a)\textstyle\sum_{k=2}^{m}\pi_k + r(1-p_s)\pi_m,$$

and $\sum_{k=0}^{m}\pi_k = 1$, we find the desired solution.

APPENDIX C
PROOF OF LEMMA 1

First, let us prove that for a fixed p_s, we have $\frac{\partial\lambda}{\partial r} \ge 0$.

$$\frac{\partial\lambda}{\partial r} = \lambda_0\,\frac{a/r}{a+r}\,\frac{\beta^{m+1} - m(\beta-1) - \beta}{\beta^m(\beta-1)^2},$$

where $\beta \triangleq \rho^{-1} \ge 1$. Notice that the expression

$$\begin{aligned}\beta^{m+1} &- m(\beta-1) - \beta\\ &= (1 + (\beta-1))^{m+1} - (m+1)(\beta-1) - 1\\ &= \sum_{k=2}^{m+1}\binom{m+1}{k}(\beta-1)^k \ge 0.\end{aligned}$$

Therefore, $\frac{\partial\lambda}{\partial r} \ge 0$. Then, $\lambda_0 \le \lambda \le \lim_{r\to\infty}\lambda$, from which we conclude the proof through the calculation of the limit.

APPENDIX D
PROOF OF PROPOSITION 2

Using the Markov chain we can show that the moment generating the function of L and its expectation is given by

$$\begin{aligned}M_L(t) &= \frac{1}{\mathbb{P}(D=\infty)}\Bigg(\sum_{k=1}^{m}(1-p_s-\rho)\rho^{k-1}\left(1-\tfrac{t}{r+a}\right)^{-k}\\ &\quad + (1-p_s)\rho^m\left(1-\tfrac{t}{r+a}\right)^{-m}\left(1-\tfrac{t}{a}\right)^{-1}\Bigg),\\ \mathbb{E}[L] &= \tfrac{\mathrm{d}}{\mathrm{d}t}M_L(t)\big|_{t=0} \qquad (11)\\ &= \tfrac{1}{r+a}\left(\tfrac{1}{1-\rho} - \tfrac{\rho^m(1-p_s(1+m\rho))}{1-\rho-p_s(1-\rho^{m+1})}\right) + \tfrac{1}{a}(1-p_s)\rho^m.\end{aligned}$$

Using (8), (9) and taking the expectation gives us

$$\mathbb{E}[A] = \mathbb{E}[Y] + \mathbb{E}[T] = \tfrac{1}{a} + 2\,\overline{D} + \mathbb{E}[K]\mathbb{E}[L].$$

Then, using (6), (7), and (11) concludes the proof.

APPENDIX E
PROOF OF PROPOSITION 3

If $p_s\lambda < \lambda_0 < \lambda$, then there exist an infinite number of pairs m, r that we can choose to reach the desired traffic λ, because for each integer m that satisfies Lemma 1 there exists a unique $r \in \mathbb{R}_+$ that achieves the traffic λ, by the fact that $\partial\lambda/\partial r > 0$ from the proof of Lemma 1 (Appendix C).

After some tedious manipulations using the definition of ρ along with (3), we can rewrite (7) and (10) as

$$\overline{D} = \tfrac{1-p_s-\rho}{a\,(1-p_s)(1-\rho)}\left(\rho - (\Upsilon+\rho-1)\tfrac{\ln\left(\frac{\Upsilon}{\Upsilon+\rho-1}\right)}{\ln(1/\rho)}\right),$$
$$\begin{aligned}\mathbb{E}[A] = \tfrac{1}{a}\Bigg[1 &+ (\Upsilon - p_s)\tfrac{1-p_s}{p_s}\tfrac{\Upsilon+\rho-1}{\rho\Upsilon} + \tfrac{1-p_s-\rho}{\rho\,(1-p_s)}\times\\ &(1-\Upsilon)\left(\tfrac{1-p_s}{p_s} + \tfrac{\rho}{1-\rho} - \tfrac{\rho\,(\Upsilon+\rho-1)\ln\left(\frac{\rho\Upsilon}{\Upsilon+\rho-1}\right)}{(1-\rho)(1-\Upsilon)\ln(1/\rho)}\right)\Bigg],\end{aligned}$$

where $\Upsilon \triangleq \lambda_0/\lambda$ is fixed. Further, from Lemma 1 we can state that $0 \le 1-\Upsilon \le \rho \le 1-p_s \le 1$. Then, we can show that $\frac{\partial\overline{D}}{\partial\rho} \le 0$ and $\frac{\partial\mathbb{E}[A]}{\partial\rho} \le 0$ as $\rho \in (1-\Upsilon, 1-p_s)$. We know that m decreases with ρ for a fixed Υ (see (3)), then to achieve the smallest mean delay and mean PAoI, we must use the smallest m. This concludes the proof.

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